

Fairness Issues and Measures in Wireless Networks: A Survey

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Abstract: The performance of resource allocation algorithms for wireless networks is measured by various parameters. One of the parameters is the 'fairness of allocation' which is a widely studied topic. A number of metrics have been defined in literature for measuring fairness. Jain's Index is the most popularly used metric. A survey of these metrics and issues therein has been carried out in this paper. Possible methods to address these issues have also been proposed.

Keywords: Wireless Networks, Resource Allocation, Fairness Metrics

I. Introduction

In a single resource system, the resource is shared by multiple users of the system. In case of wireless networks, the Resource Allocation Algorithms (RAAs) assign to each user a share of the resource. Widely used RAA such as Maximum Throughput (MT), Modified Largest Weighted Delay First (M-LWDF), Proportional Fair (PF), Exponential Rule (EXP-RULE), Logarithmic Rule (LOG-RULE), Frame Level Scheduler (FLS) and a modified version of PF i.e. Exponential Proportional Fair (EXP-PF) can be found in [1]-[9]. Fairness of these algorithms is determined using the fairness metrics such as Proportional Fairness, Max-Min, Jain's Index and Entropy. Some issues associated with these measures, which might reduce their efficacy, have been described in this paper.

In section II of this paper, the concept of fairness in wireless networks is discussed. The related literature review on fairness is described in section III. Issues relating to fairness models are discussed in section IV and section V concludes the paper.

II. Fairness In Wireless Networks

Fairness as a general English term refers to impartiality, justice and satisfaction of individuals [10]. In wireless networks the process of resource allocation defines fairness. Fairness strategies have been designed to distribute resources 'fairly' to individual users of the system. Unfairness of allocation among individuals of the system has not been dealt with in majority of the fairness designs. Mostly fairness is defined for an individual in terms of the amount of resource actually received as compared to a value defined by the system designer. Along with this, a system level of fairness is also defined which gives the value of fairness for the entire wireless network system.

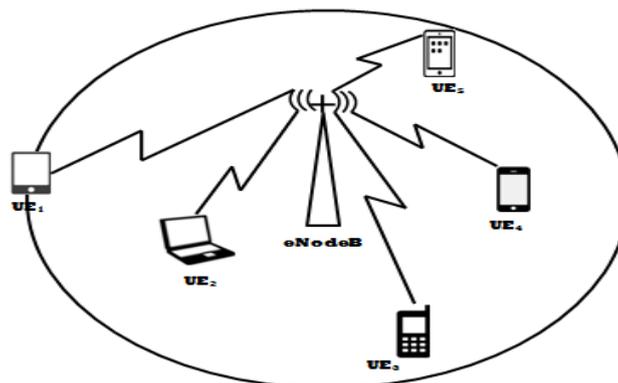


Figure 1: A simple LTE (wireless network) scenario comprising of a single cell of 5 UEs

Fairness in a wireless network can be ensured by allocating a 'fair amount' of resource (bandwidth) to each user. In addition, each user expects a 'fair level' of Quality of Service (QoS) from the network. This can be further explained by using a simple example of a wireless cellular network (LTE network) as shown in Fig. 1. User Equipments (UE) 1, 2, 3, 4 and 5 are devices that are serviced by the base station (eNodeB). In this scenario, each UE requests for services such as VoIP, video buffering, audio call, email or file transfer. Significance of fairness in wireless networks can be explained by exploring fairness issues in such a scenario. For instance, the UEs should get a 'fair chance' to access the network resource; QoS requirements of the UEs should be 'fairly satisfied'.

- (i) *fair chance of allocation*: each user equipment must get a certain amount of resource. If fairness of access to resource is not ensured then a UE might starve due to no allocation.
- (ii) *fairly satisfied QoS requirements*: the QoS requirements of a UE (depending on the type of service requested) should be fulfilled to the extent which ensures satisfaction of service by the UE.

III. Fairness Measurements

Fairness can be defined by quantitative measures as well as qualitative measures. The quantitative measures give a real number representation for fairness. This representation allows for comparative analysis of fairness values for individuals of the system. Most widely used quantitative measures are Jain’s Index and Entropy. There are some qualitative measures that cannot provide a real number representation of fairness but can ensure fairness. Two such measures are Max-Min fairness and Proportional Fairness. Fig. 2 shows the symbols along with their meanings used in defining fairness measures.

- 1) $\mathbf{X} = \langle x_1, x_2 \dots x_n \rangle$: wireless network resource,
where each $x_i \in \mathbb{R}^+$ represents the amount of resource allocated to a user i
- 2) n : number of users of the network
- 3) Operations on x_i :
 - (i) $\sum x_i \mid i = 1, 2 \dots n$: sum of metric values of all users;
 $0 < \sum x_i \leq \kappa$; where κ is the total amount of the resource
 - (ii) $\sum x_i^2$: sum of square of metric values of all users
 - (iii) $x_j \geq x_i \mid j \neq i$: amount of resource \mathbf{X} allocated to user j is more than that allocated to another user i
 - (iv) $x_j \leq x_i \mid j \neq i$: amount of resource \mathbf{X} allocated to user j is less than that allocated to another user i
- 4) $f(\mathbf{X})$: representation of Jain’s Fairness Index
- 5) $H(\mathbf{P})$: representation of Entropy

Figure 2: Symbols used in defining fairness measures

a) Quantitative Measures

Quantitative measures can be represented by a general fairness function $f(\mathbf{X})$ and its properties are given below:

- i. Real-valued : $f(\mathbf{X})$ is a real-valued function defined by $f(\mathbf{X}) : \mathbb{R}_n^+ \rightarrow \mathbb{R}^+$
- ii. Continuity : $f(\mathbf{X})$ is defined for its entire domain (\mathbb{R}_n^+)
- iii. Scalability : $f(\mathbf{X})$ is independent of the number of users n and scales with n
- iv. Bounded function : $f(\mathbf{X})$ is a finite function and can easily mapped on to $[0,1]$
- v. Independence of metric and scale
- vi. Extensibility of $f(\mathbf{X})$ to multi-resources case
- vii. Sensitivity of $f(\mathbf{X})$ to variation of \mathbf{X} : $f(\mathbf{X})$ should be sensitive enough

The two popularly used measures to determine fairness quantitatively are explained in the succeeding subsections. Table I shows the properties of these fairness measures and Table II shows different Jain’s Index and Entropy values for different network allocations for scenario given in Fig. 1.

Jain’s Fairness Index

Proposed by R. K. Jain, Jain’s (fairness) Index [11] is the most widely used fairness measure. It describes fairness index in terms of equality of user allocation \mathbf{x} . The greater the equality of allocation among the n users, the closer is the fairness index to 1. This index nears 0 if the allocation is favoured for a selected few users. Jain’s fairness index can be defined as shown in (1):

$$f(\mathbf{X}) = \frac{[\sum_{i=1}^n x_i]^2}{n \cdot \sum_{i=1}^n x_i^2}; \quad x_i \geq 0 \quad \dots (1)$$

The Jain’s Index can also be written in terms of ‘average of perceived fairness of all n users’. To achieve this, a fair allocation mark x_f is computed in (2.1). Each user compares its allocation x_i with x_f to determine the amount of fairness/ unfairness observed by it. Fair allocation mark is determined by:

$$x_f = \frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n x_i} \quad \dots (2.1)$$

For each user, the algorithm is only x_i/x_f fair. The overall fairness thus can be written as:

$$f(\mathbf{X}) = \frac{1}{n} \sum_{i=1}^n \frac{x_i}{x_f} \quad \dots (2.2)$$

The ‘perceived discrimination’ of user i is computed as $(x_f - x_i)/x_f$. The discrimination index is therefore defined as:

$$\text{Discrimination Index} = \frac{1}{n} \sum_{i=1}^n \frac{x_f - x_i}{x_f} \quad \dots (3)$$

As can be seen, (3) also states that among n users of the system, not all users can be discriminated. Therefore the function $f(x)$ is bounded by $[1/n, 1]$.

Entropy

Entropy function introduced by Shannon [12] is also used as a fairness measure. Its description is given below. There are n individuals and the proportions of resource distributed to an individual i (p_i) is defined as

$$P = (p_1, p_2, \dots, p_n)$$

Where each proportion p_i is defined as:

$$p_i = \frac{x_i}{\sum_{i=1}^n x_i} \quad \dots (4)$$

The entropy of this distribution is defined by $H(P) = H(p_1, p_2, \dots, p_n)$ and is given by:

$$H(P) = - \sum_{i=1}^n (p_i \log_2 p_i); \quad 0 \leq p_i \leq 1 \text{ and } \sum_{i=1}^n p_i = 1 \quad \dots (5)$$

When used as a fairness measure, $H(P)$ is similar to $f(X)$. Only the absolute resource values of X ($x_1, x_2 \dots x_n$) are replaced by resource proportions P ($p_1, p_2 \dots p_n$). Larger the value of $H(P)$, fairer is the allocation.

b) Qualitative Measures

Qualitative measures of fairness can be written in the form shown below:

If (condition satisfied) **then** (algorithm is ‘fair’)

Since these measures cannot provide real number representation of fairness, they can be used after any of the quantitative measures to ensure fairness of allocation. The two popularly used measures that describe fairness qualitatively are explained in the succeeding subsections.

Max-Min Fairness

An algorithm is said to have achieved max-min fairness [13] if the following condition is satisfied

x_i cannot be increased to $\{x_i + \eta_j\}$; (η_j is a portion of x_j) without decreasing x_j ($x_j \leq x_i$)

This condition states that the allocation of resource X to a user i (x_i) cannot be increased without decreasing the allocation of another user j (x_j) which is already less than x_i . It ensures that the allocation $\langle x_1, x_2 \dots x_n \rangle$ is the ‘fairest possible allocation’ of the resource X to n users.

Similar to this, min-max fairness is also defined as

x_i cannot be decreased to $\{x_i - \eta_j\}$; (η_j is a portion of x_j) without increasing x_j ($x_j \geq x_i$)

This condition states that the allocation of resource X to a user i (x_i) cannot be decreased without increasing the allocation of another user j ($x_j \geq x_i$) while maintaining the feasibility. It ensures that the allocation $\langle x_1, x_2 \dots x_n \rangle$ is the ‘fairest possible allocation’ of the resource X to n users.

An algorithm whose fairness is ensured by max-min as *perfect fairness*, allocates equal share of the resource to every individual.

Proportional Fairness

The concept of proportional fairness was first introduced by [14]. As a fairness measure, it ensures fairness of allocation of resources to a user in a multi resource system. For a multi-resource system with m resources with each resource j having a finite capacity C_j and n users, x_{jk} is the amount of resource j allocated to user k . The allocation of finite capacity resource j to all individuals of the system can be written as

$$X_j = \langle x_{j1}, x_{j2} \dots x_{jn} \rangle$$

Thus an individual’s allocation written as $x_i = \langle x_{1i}, x_{2i} \dots x_{mi} \rangle$ is said to be proportionally fair if it satisfies the following conditions:

(i) Real-valued allocation of resource j to user i : $x_{ji} \geq 0$

(ii) Feasible allocation of a resource to all users:

$$\sum_{i=1}^n x_{ji} \leq C_j$$

(iii) Fairest allocation of resource to a user:

$$\sum_{i=1}^n \frac{x_{ji}^* - x_{ji}}{x_{ji}} \leq 0$$

where x_{ji}^* is the allocation of resource j to user i other than x_{ji} .

Table I: Properties Of Quantitative Fairness Measures

Property	Jain's Index	Entropy
Boundedness	$\frac{1}{n} \leq f(\mathbf{X}) \leq 1$	$0 \leq p_i \leq 1$
Scalability	$f(\mathbf{X}) = \frac{k}{n}$ k = no. of favoured users each allocated c/k of the resource c = finite capacity of the resource	$H(\mathbf{P}) = -\sum_{i=1}^k \frac{1}{k} \cdot \ln \frac{1}{k}$
Independence of metric & scale	$f(\mathbf{X}) = \frac{1}{1+COV^*}$	Similar to $f(\mathbf{X})$
Continuity	See table II (case II) where a small change in the allocation is reflected in the fairness index	
Multi-resource case	\mathbf{X} in $f(\mathbf{X})$ can be assigned as a combination of throughput, input load and normalized throughput [15]	Similar to $f(\mathbf{X})$

Table II: Examples of Jain's Fairness Index and Entropy

x_i	Case I	Case II		Case III		Case IV
		x_i	x_i'	x_i	x_i'	
x_A	0%	5%	5%	10%	10%	20%
x_B	5%	40%	40%	30%	28%	20%
x_C	30%	50%	48%	30%	30%	20%
x_D	0%	5%	5%	10%	12%	20%
x_E	65%	0%	2%	20%	20%	20%
$f(\mathbf{X})$	0.3883	0.4819	0.5053	0.8333		1
$H(\mathbf{P})$	---	---	0.8589	1.5048	1.5241	1.609
Variance (V) = $(x_i - x_m)^2 / (n-1)$				100	10000	
Standard Deviation (SD) = \sqrt{V}				10	100	
*Coefficient of Variation (COV) = SD / x_m				0.5	0.5	

IV. Issues

The above fairness measures are widely used for computing fairness of allocation in wireless networks. There are certain issues associated with them that need to be addressed in order to improve their performance. These issues are described below.

(i) Complete information of allocation required

The fairness measures described above require complete information of allocation. It means these measures can only compute fairness values but cannot improve the fairness of allocation algorithm, as fairness is calculated only after the resources have been allocated.

(ii) Unfairly treated individuals not identified

Jain's Index computes *discrimination index* as *one minus fairness index* and also the perceived discrimination for user. The question that arises is "is unfairness simply one minus fairness". Similar is the case with Entropy, only the x_i is replaced by resource proportion p_i .

To determine unfairness in such a case some parameter needs to be defined that shall differentiate fair allocations from the unfair allocations. The allocations that are treated as unfair shall then add to the discrimination index.

(iii) Over allocation of resource

If a user is allocated more resource than required, then is this over allocation acceptable by other users as the total resource is of finite capacity. In such a case it is required to be checked if over allocation to a user causes *starvation* of another user(s). If so then some checks are required to ensure that over allocation of resource is considered as an *unfair allocation* and add to the *discrimination index* measure.

(iv) Entropy not defined if no allocation to a user

In case of no allocation of resource to a user ($x_i=0$), $H(\mathbf{P})$ is undefined. Thus entropy can be used as a fairness measure only if all the users are allocated some portion of the resource. However, due to finite capacity of the resource and/ or over allocation of the resource one or more individual of the system might starve in which case entropy cannot completely measure fairness of the system.

V. Conclusion

An overview of fairness in wireless networks has been provided in this paper. The popularly used fairness measures and issues associated with them have been discussed. Proposing a comprehensive solution to these issues is out of the scope of this paper. However, possible methods to overcome these issues have been briefly stated. To resolve these issues either modification of existing measures is required or the development of a novel measure is required. Various fairness models have also been proposed in the literature, but they are generalizations of these existing measures. Also they are very complex and difficult to implement. Hence there is scope for development of a fairness model that is less complex and easy to implement.

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